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HEAT TRANSFER FROM A POROUS CIRCULAR CYLINDER IMMERSED IN A MOVING STREAM

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NOMENCLATURE

a,	radius of the porous cylinder;
a_n, b_n, c_n, d_n,	coefficients in the asymptotic expansions of the temperature field;
g_n,	outer expansion of velocity;
k,	dimensionless permeability;
k',	physical permeability;
N,	Nusselt number;
$\mathbf{q}$,	dimensionless velocity vector, q'/U_∞;
r,	radial coordinate in physical space normalized with a;
R,	Reynolds number, $U_\infty a/\nu$;
t_0/t_∞ :	temperature at the wall and at infinity, respectively;
T,	dimensionless temperature;
x,	inner variable, x'/a;
x',	physical distance.

Greek symbols

δ_n, Δ_n,	asymptotic sequences in the temperature expansions;
ε,	asymptotic sequence in the velocity expansion;
ρ,	outer variable, Rr;
σ,	Prandtl number;
ν,	kinematic viscosity.

1. INTRODUCTION

HEAT transfer in porous media is of growing interest because of applications in geometrical reservoirs and in thermal recovery processes. The theoretical and experimental studies concerning heat transfer from impervious bodies, in a uniform fluid stream, have been summarized in detail by Heiber and Gebhart [1]. However, there is neither experimental nor analytical information available for heat transfer between a permeable cylinder and a moving stream in which it is immersed. The present results are the first calculation of heat transfer from a porous permeable cylinder. The flow field was determined by Shi and Braden [2].

The effect of permeability on the steady heat transfer from an isothermally heated porous circular cylinder is calculated under the assumption of a Reynolds number, $R < 1$. The matched asymptotic method of solution is used for both conditions analyzed here, that is, a moderate and a large Prandtl number. The velocity field used in the energy equation was that determined in ref. [2]. The numerical values of the average Nusselt number are determined for different levels of permeability k.

2. FORMULATION

These calculations concern a long heated isothermal porous circular cylinder at t_0, placed normal to a uniform stream at t_∞, moving at the speed U_∞. The x-axis is taken in the flow direction of the uniform stream. The properties of the fluid are taken as constant. The viscous dissipation, pressure energy and buoyancy terms are neglected. The resulting energy equation in the outside Newtonian region is

$$\nabla_r^2 = \sigma R \mathbf{q} \cdot \text{grad } T \tag{1a}$$

with the appropriate boundary conditions

$$T = 1 \text{ at } r = 1, \tag{1b}$$

$$T \to 0 \text{ as } r \to \infty \tag{1c}$$

where T is the dimensionless temperature $T = (t - t_\infty)/(t_0 - t_\infty)$, t_0 and t_∞ are the temperatures at the surface and at great distance, respectively, r is the radial coordinate non-dimensionalized with respect to the radius a and $R = U_\infty a/\nu$.

$$\nabla_r^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}.$$

The velocity-field and resulting streamlines in the fluid are first calculated from the results of Shi and Braden [2]. The velocity is

$$\mathbf{q} = i + \varepsilon g_1(x_i) - k_0 \nabla\left(\frac{x}{r^2}\right) + O(\varepsilon^2)$$

where i is the unit vector along the x-axis and

$$\varepsilon = \text{asymptotic sequence} = \frac{1}{k_0 + \ln[4/(\gamma R)]}$$

Here $k_0 = 1/2 + k$ is the dimensionless permeability $k = k'/a^2$ and $\ln \gamma =$ Euler's constant $\simeq 0.577$. The function $g_1(x_i)$ is

$$g_1(x_i) = -2i \exp\left(\frac{\rho\cos\theta}{2}\right)k_0(1/2\rho) + 2\nabla\left[\exp\left(\frac{\rho\cos\theta}{2}\right) + k_0(1/2\rho) + \ln\rho\right]$$

where $\rho = Rr \equiv$ outer (or Oseen) variable and θ is the angular coordinate measured from the downstream direction. The above expression of the velocity distribution is uniformly valid everywhere outside the cylinder. Inside the porous cylinder, it is given by

$$\mathbf{q} = 2(\varepsilon + a_2\varepsilon^3)\, ki + O(\varepsilon^4)$$

where $a_2 = -0.87$.

On the basis of the above velocity field, the flow patterns in and around the permeable cylinder in the Stokes region are computed in detail. The particular streamline which is composed of the two radial lines $\theta = 0$ and $\theta = \pi$, is denoted by $\psi = 0$. Then, in the external Stokes region, the general equation of the streamlines, to the order of the present analysis, is expressed as

$$\psi(r, \theta) = r\varepsilon[i + a_2\varepsilon^2][k_0 - 1 + \ln r + (k_0/r^2)] \sin\theta = \text{const.}$$

Interior to the cylinder, in the Darcy region, it is given by

$$\psi(r, \theta) = 2r\varepsilon(1 + a_2\varepsilon^2) k \sin\theta = \text{const.}$$

In the Oseen region the stream function is that of a uniform stream perturbed by terms of $[O(\ln R)^{-1}]$. In the half-field of flow above (or under) the central streamline, $\psi = 0$, the last streamline "touching" the cylinder is found to be

$$\psi = 2k\varepsilon(1 + a_2\varepsilon^2).$$

The nature of the flow is visualized in Fig. 1, where various streamlines have been plotted, for $R = 0.2$ and $k = 2.5$. It is seen that the flow is perpendicular at the surface of the permeable cylinder, a consequence of the zero tangential component condition. Then, inside, streamlines are simply parallel to the x-axis. The discontinuity of the first derivative of the stream function at the boundary is due to the discontinuity of the tangential velocity. This arises due to the lower order of the empirical Darcy's law governing the flow inside the porous media, compared to the Navier–Stokes equations which are used outside of the cylinder.

In the analysis of the flow of a viscous fluid past a porous and permeable body, the problem of specifying the appropriate boundary conditions at the interface has not yet been optimally resolved. Most workers consider the continuity of only the normal component of velocity [3–7]. Some, in studies of thermal convective instability in porous media [8, 9], have included the inertia term $q.\nabla\mathbf{q}$. This term raises the order of the Darcy equation. However, as pointed out by Beck [10], this leads to an underspecified system, if only the normal component of the velocity on the boundary is prescribed. On the other hand, the system becomes overspecified if the tangential velocity is also prescribed. This ambiguity of appropriate boundary conditions can only be resolved by further experimentation in such flows.

3. RESULTING TEMPERATURE DISTRIBUTION

Heat transfer for both moderate and high Prandtl number fluids has been analyzed. For the latter, the analysis is highly simplified by making $\sigma = R^{-\alpha}$ where α is then subsequently determined from the Prandtl and Reynolds numbers. From the analysis of Hieber and Gebhart [1], it is found that, in the Oseen thermal region, the leading term of the velocity field is $(1 - \alpha)\, U_\infty$. This yields an "effective Reynolds Number" $(1 - \alpha) R$, where $0 < \alpha < 1$. For $\sigma \sim 1$, the temperature field is assumed to be represented by the following inner and outer expansions:

$$T(r, \theta) = \sum_{n=0}^{\infty} \delta_n(R)\, T_n(r, \theta), R \downarrow 0, r \text{ fixed} \quad (2)$$

$$T(\rho, \theta) = \sum_{n=0}^{\infty} \Delta_n(R)\, T_n(\rho, \theta)\; R \downarrow 0, \rho \text{ fixed} \quad (3)$$

where $\lim_{R\to 0} \frac{\delta_{n+1}}{\delta_n} = 0$, $\lim_{R\to 0} \frac{\Delta_{n+1}}{\Delta_n} = 0$.

Substitution of the Stokes and Oseen variables into the energy equation yields to, respectively

$$\nabla r^2 T = \sigma R\left(q_r \frac{\partial T}{\partial r} + q_\theta \frac{\partial T}{r\partial\theta}\right) \quad (4)$$

and

$$\nabla\rho^2 T = \sigma\left(q_r \frac{\partial T}{\partial\rho} + q_\theta \frac{\partial T}{\rho\partial\theta}\right). \quad (5)$$

The construction of the solution is analogous to that given in ref. [1]. From the same matching considerations it is found that the structure of the temperature expansions (2) and (3) is similar to the corresponding ones for the non-porous cylinder

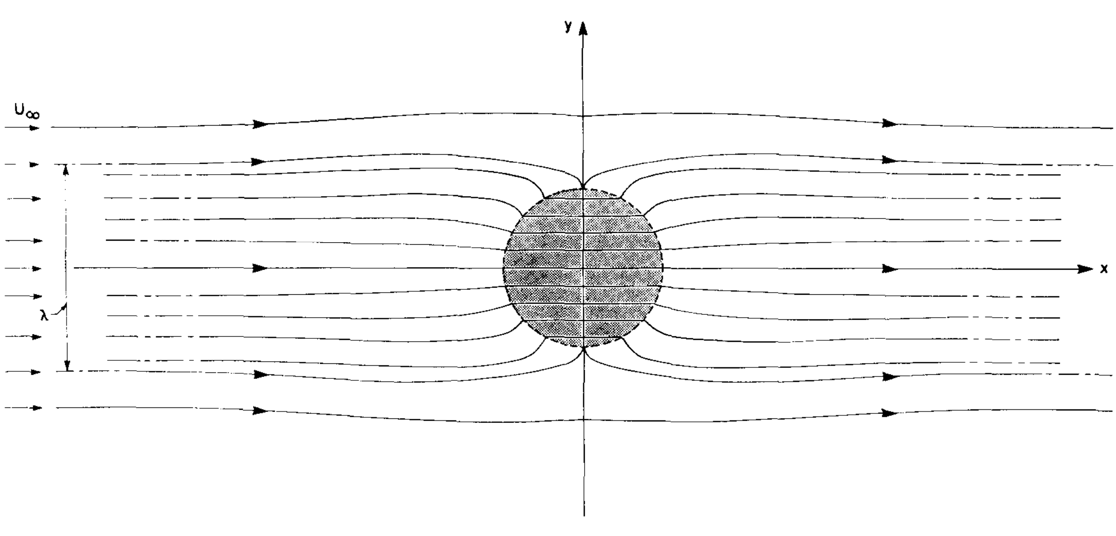

FIG. 1. Calculated streamlines past a porous circular cylinder of permeability $k = 2.5$, at Reynolds number $R = 0.2$. In the half-field of flow above (or under) the central streamline $\psi = 0$, they correspond respectively to $\psi = 0.20, 0.40, 0.60, 0.80, 0.895$ and 1.25.

problem. The inner expansion of $T(r, \theta)$ is expressed as

$$T(r, \theta) \sim 1 - \left[\Delta - \sum_{3}^{\infty} c_n(\sigma, k)\, \Delta^n\right] \quad \ln r,\ r \text{ fixed},\ R \downarrow 0 \quad (6)$$

where

$$\Delta = \left(\ln \frac{4}{\sigma\gamma R}\right)^{-1} \quad (7)$$

and

$$c_n(\sigma, k) = \text{constants},$$

functions of the permeability, to be determined (cf. Appendix 1). The outer expansion of $T(\rho, \theta)$ is given as

$$T(\rho, \theta) \simeq \Delta T_0 + \Delta^2 T_1 + O(\ln R)^{-3} \quad \rho \text{ fixed},\ R \downarrow 0 \quad (8)$$

where

$$T_0 = \exp\,(\sigma\rho \cos\theta)/2\ k_0\,(\sigma\rho)/2.$$

Because of the method of matched asymptotic expansions, T_1 need not be expressed explicity. Only its behavior as ρ goes to 0, that is, in the matching region, must be obtained. This permits evaluation of the constant $c_3(\sigma, k)$.

For the high Prandtl number fluid, it may be shown that the new "Oseen" variable is

$$\hat{\rho} = rR^{(1-\alpha)}.$$

Using the same expressions for the temperature as in equations (2) and (3), the resulting analysis parallels that for σ near 1. The temperature distributions in the thermal Oseen and Stokes regions are, respectively,

$$T(\hat{\rho}, \theta) \simeq \phi\,\hat{T}_0 + \phi^2\,\hat{T}_1 + O(\phi^3),\ \hat{\rho} \text{ fixed},\ \sigma = R^{-\alpha},\ R \downarrow 0 \quad (9)$$

$$T(r, \theta) \simeq 1 - \left(\phi - \sum_{3}^{\infty} d_n(\alpha, k)\,\phi^n\right) \quad \ln r,\ r \text{ fixed},\ \sigma = R^{-\alpha},\ R \downarrow 0 \quad (10)$$

where

$$\hat{T}_0 = \exp\,(1/2\ \sigma\rho\cos\theta)\ k_0\,[1/2\,(1-\alpha)\rho],$$

$$\phi = \ln\left[\frac{4}{(1-\alpha)R\gamma\sigma}^{-1}\right].$$

The coefficients $d_n\,(\sigma, k)$ are functions of the permeability k. (cf. Appendix 1).

4. AVERAGE NUSSELT NUMBER

The main interest here is the average Nusselt number N. It is obtained from equation (6), for a moderate Prandtl number, and from equation (10) for a high Prandtl number. Recall that

$$N = -\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{\partial T}{\partial r}\right)_{r=1} d\theta.$$

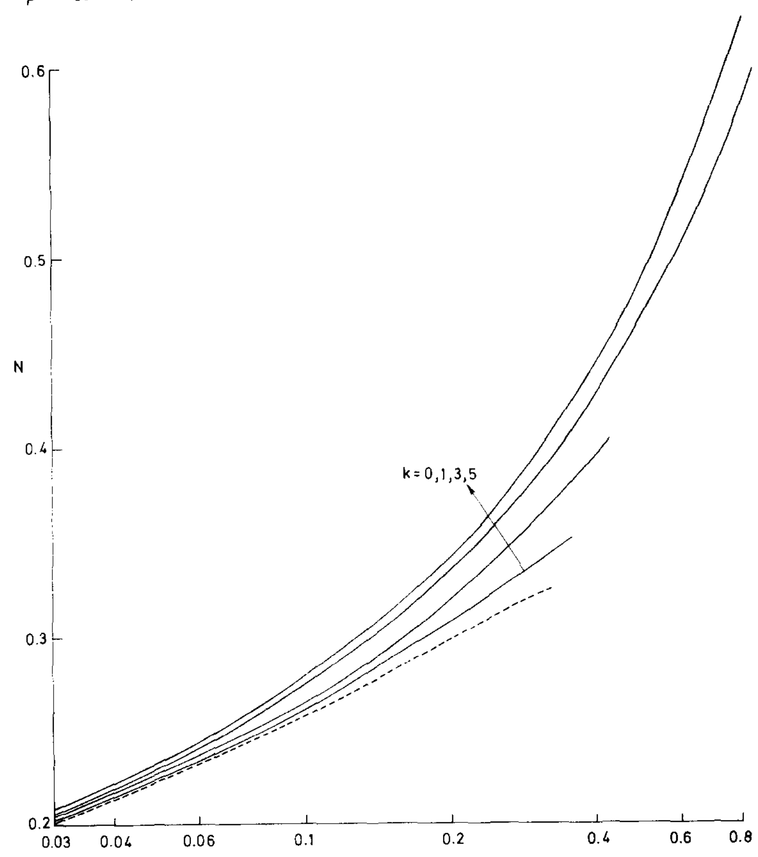

FIG. 2. Effect of permeability k on Nusselt number N as a function of Reynolds number R for a porous cylinder in air ($\sigma = 0.72$). The dashed line curve corresponds to the results of Hieber and Gebhart for nonporous cylinder.

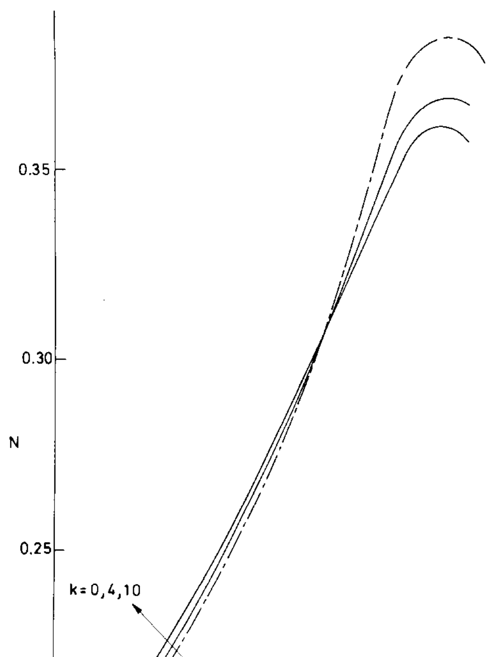

FIG. 3. Effect of permeability k on Nusselt number N for a porous cylinder in water ($\sigma = 6.82$).

For moderate σ

$$N \simeq \frac{1}{\ln[4/(R\gamma\sigma)]}\left[1 - \frac{a_3(\sigma)}{k_0 + \ln[4/(\gamma\sigma R)]}\right] + O(\ln R)^{-4}\, R \downarrow 0. \quad (11)$$

For high σ

$$N \simeq \frac{1}{\ln[4/\gamma\sigma R(1-\alpha)]}$$

$$\left[1 - \frac{b_3(\chi)}{k_0 + [\ln(4/\gamma\sigma(1-\alpha)R)][\ln(4/\gamma\sigma(1-\alpha)R]}\right] + O(\ln R)^{-4},\ R \downarrow 0 \quad (12)$$

where $a_3(\sigma)$ and $b_3(\alpha)$ are the same as obtained in ref. [1].

The resulting effect of permeability on the forced heat convection from a porous circular cylinder in air ($\sigma = 0.72$) is shown in Fig. 2. It is observed that N increases as the permeability k increases. However, for R very small the effect is seen to become very small. High Prandtl number results are seen in Fig. 3, for $\sigma = 6.82$. The effect of permeability is much less over the whole range.

No experimental data has been found for such heat transfer. By making k zero in equations (11) and (12), the results of Hieber and Gebhart should be recovered. The resulting disagreement (Fig. 2) is due to the fact that they have approximated $1/2 + \ln[4/(\gamma\sigma R)]$, as $\ln[4/(\gamma\sigma R)]$ in the determination of $T_1(\rho, \theta)$.

5. CONCLUSIONS

The above results show the considerable effects on heat transfer of submerged surface permeability. These arise even in the highly viscous flows over cylinders of small diameter. Such a penetration would appreciably increase, for example, the melting rate of a rod of ice immersed in flowing water at a temperature above the melting point. Such a multiple region flow would arise with any fragmented immersed solid. In a melting process, the isothermal condition assumed above is appropriate.

The Nusselt number curves for several values of k, in Figs. 2 and 3, indicate that the value of N for water is again much larger than that for air even with a permeable surface. The Nusselt number also increases with increasing permeability, in both fluids. This results from the increased thinning of the thermal region and steepening of the temperature gradient, because of the flow of fluid into the cylinder. The later crossing seen at higher R, for $\sigma = 6.82$, apparently arises from a complicated interaction of several effects.

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APPENDIX 1

Values of $c_3(\sigma, k)$ and $d_3(\sigma, k)$

Equation for T_1 is written as

$$\nabla_\rho^2 T_1 = i.\ \text{grad}\ T_1 + \frac{1}{1+\Delta k_0} g_1(x_i) - k_0 \nabla(x/r^2) \cdot \text{grad}\ T_0$$

we let $T_1(\rho, \theta) = \exp[(\sigma\rho\cos\theta)/2]\, F(\rho, \theta)$.

The equation becomes

$$(\nabla_\rho^2 - 1/4\sigma^2)F = f(\rho, \theta)$$

where

$$f(\rho,\theta) = \frac{\sigma^2}{1+\Delta k_0}\cos\theta\, k_0\,(\sigma\rho/2 - k_1)$$

$$+ 1/2\frac{\sigma^2}{(1+\Delta k_0)}\exp\frac{\rho\cos\theta}{2}k_1(\rho/2)\,k_1(\sigma\rho/2)$$

$$-\cos\theta\, k_0\,(1/2\,\rho)\,k_1(\sigma\rho/2) - k_0(\rho/2)\,k_0(\sigma\rho/2)$$

$$+\frac{2}{(1+\Delta k_0)}\cos\theta\, k_0(\sigma\rho/2) - k_1(\sigma\rho/2)$$

$$+\frac{k_0}{(1+\Delta k_0)}(R/\rho)^2\cos 2\,\theta\; k_0\;(\sigma\rho/2)$$

$$-\cos\theta\, k_1\,(\sigma\rho/2).$$

Solving the equation by Green's function method, we have the appropriate Green's function

$$G(\bar{\rho},\bar{\rho}') = -\frac{\pi k_0}{2}\left[\frac{\sigma|\bar{\rho}-\bar{\rho}'}{2}\right]$$

therefore

$$T_1(\rho=0) = F(\rho,0) = \int_0^{\infty}\int_0^{2\pi} G(0,\rho')\,f(\rho',\theta')\,\rho'\mathrm{d}\rho'\mathrm{d}\theta'.$$

Note that the contribution of the term in $f(\rho',\theta')$

$$\frac{k_0}{1+\Delta k_0}(R/\rho')^2\cos 2\theta' k_0\,(1/2\,\sigma\rho')$$

$$-\cos\theta'\,k_1\,(1/2\,\sigma\rho')\,k_0\,(1/2\,\sigma\rho')$$

to the integral is zero.

After rearrangement, it is found that

$$c_3(\sigma,k) = \frac{a_3(\sigma)}{1+\Delta k_0}$$

where $a_3(\sigma = 0.72) = 1.38$ (ref. [1]).

Similarly, for the high Prandtl number theory,

$$d_3(\sigma,k) = \frac{b_3(\alpha)}{1+\Delta k_0}.$$

$b_3(\alpha)$ are computed in ref. [1].

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FLOW PAST A SUDDENLY COOLED VERTICAL FLAT SURFACE IN A SATURATED POROUS MEDIUM

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NOMENCLATURE

- Q, heat transfer parameter;
- t, time;
- T_w, wall temperature (for $t < 0$);
- T_0, ambient temperature;
- u, Darcy's law velocity in the x-direction;
- v, Darcy's law velocity in the y-direction;
- x, (non-dimensional) coordinate along the wall;
- y, (non-dimensional) coordinate normal to the wall.

Greek symbols

- η, $y/x^{1/2}$;
- θ, (non-dimensional) temperature;
- ψ, stream function;
- ζ, $y/2t^{1/2}$;
- τ, t/x.

INTRODUCTION

HEAT transfer from a surface embedded in a porous medium through which a liquid is flowing is of great practical importance in many branches of engineering. For example, convective flows in porous medium are of considerable interest because of the present and potential use of geothermal energy for power production. The basic theory and much of the previous work in this area is to be found in an extensive review article [1]. Of particular interest in this context are flows near impermeable surfaces (intrusives or boundaries of the porous medium) at high Rayleigh numbers, where the boundary-layer approximations can be made. The basic solution for a vertical flat surface has been given [2] and was later extended to a boundary of arbitrary shape [3]. The boundary layer studies described [1] have all been concerned with steady flow configurations. No work has yet been done on how such flows could be set up from some given initial state. The purpose of this paper is to present solutions of the